The Chandler wobble is a phantom

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In this paper we have proved that the assertion on the existence of residual motion of the Earth rotation axis within the Earth is a result of erroneous interpretation of the zenith distance (angle) measurements with instruments using as a reference the plumb-in line or artificial horizon. The functional relationship between the plumb-in line deviation and Moon's perigee position has been established for an arbitrary point on the rotating Earth surface. This relationship manifests itself as variations in the gravitational acceleration vector direction and magnitude at the observation point or, in other words, as periodical deviations of the plumb-in line (local vertical). The results of our work show that it is necessary to revise some postulates of metrology, gravimetry, astronomy, geophysics and satellite navigation.

1. Sources of the problem

Based on observations of "variations of latitudes" from 1726 to 1890, the then astronomic community put forward a hypothesis that the latitude variation results from the rotating Earth wobbling on its rotation axis. In 1892, Chandler[1] combined the latitude variation observations collected by 17 observatories during the period from 1837 to 1891 in a united time series, processed them, and revealed that latitude variations obey a characteristic periodicity of 410–440 days [9]. The Chandler's discovery and the "variation of latidude" hypothesis did not get the necessary and sufficient experimental confirmation at that time; however, at the turn of 19th century they allowed the scientific community to come up with the following hypothesis: the Earth's rotation axis executes within the Earth residual motion with the characteristic "Chandler period". At the end of the 20th century, leading European and USA experts in the theory of the Earth's polar (rotation axis) motion and the Earth's rotation theory had to state in the paper devoted to the Chandler's discovery centenary [10] the absolute absence of results of this phenomenon investigation.

[1]Seth Carlo Chandler, Jr. (1846–1913)

2. Problem definition

The fact that the period close to the Chandler's period manifests[2] itself in variations in the gravitational acceleration on the Earth's surface leads to an assumption that Chandler's wobble in astrometry results from instability of the gravitational field at the point where the zenith distance is measured[3]. Let us consider the problem of gravitational field instability caused by the Moon as a problem of the Earth's and Moon's gravitational forces effect on a point mass located at point **A** on the Earth surface (Fig. 1). We will use an orthogonal frame of reference $\mathbf{Axyz}$ with the origin at point **A**. Assume that it is oriented so that the $\mathbf{Axy}$ plane is tangent to the Earth surface at point **A**, while the $\mathbf{Az}$ axis is directed away from the Earth. Define the

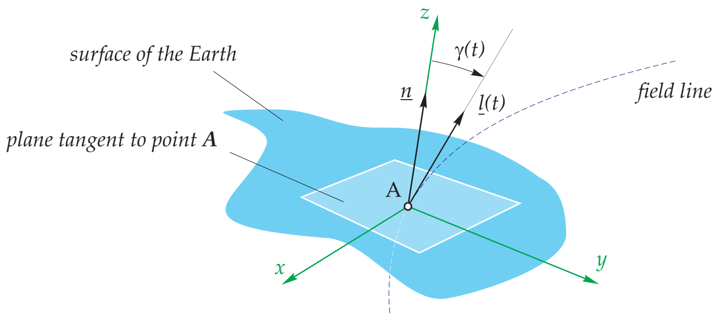

Figure 1. Angle of the gravitational field line departure at point **A**

force acting on the point mass at point **A** as a function of gradient of the Earth–Moon system gravitational field potential $\mathbf{U}(t)$:

$$\underline{f}(t) = -\nabla \mathbf{U}(t) \ . \tag{1}$$

Force $\underline{f}(t)$ that is tangent to the line of force is just that defines *the plumb-in line*. Variations in the force $\underline{f}(t)$ direction in the $\mathbf{Axyz}$ frame of reference will be followed up through vector $\underline{l}(t)$ by calculating angle $\gamma(t)$ between the fixed unit vector $\underline{n}$ ($\mathbf{Az}$ axis ort) and vector $\underline{l}(t)$ by using the cosine law:

$$\gamma(t) = \operatorname{acos}\left(\, \underline{n} \cdot \underline{l}(t) \,\right) , \quad \textit{where} \quad \underline{l}(t) = -\frac{\underline{f}(t)}{|\underline{f}(t)|} \, , \quad |\underline{l}(t)| = 1 \ . \tag{2}$$

We do not take into account other evidently existing physical phenomena since they are not critical in the problem defined here. I.e., we make the

[2]This paper does not consider results of spectral analysis of time series reflecting variations in the ocean level, atmospheric pressure and Earth gravitational acceleration.

[3]The angle between the direction to the celestial body and zenith.

problem maximally simple in order to reveal the process essence without diverting our attention to secondary factors that, however, can play a significant role under other conditions.

3. Observer

Let us consider two fixed orthogonal frames of reference $\mathbf{O}x^e y^e z^e$ and $\mathbf{O}x'y'z'$ with the common origin at point $\mathbf{O}$ (Fig. 2). Plane $\mathbf{O}x^e y^e$ belongs to the ecliptic, while plane $\mathbf{O}x'y'$ coincides with the Earth equator plane. Assume that the Earth is an ellipsoid of revolution. $\mathbf{O}z'$ is the axis of the Earth self-rotation and maximum inertia moment. $\mathbf{O}z'$ forms angle ε with the $\mathbf{O}z^e$ axis. Axes $\mathbf{O}x'$ and $\mathbf{O}x^e$ are of the same direction and are parallel to the vernal equinox line ♈. The point $\mathbf{A}$ coordinates on the Earth surface are

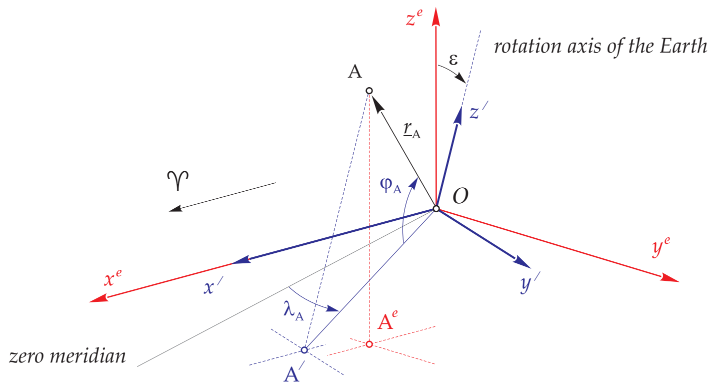

Figure 2. Observer **A** on the Earth surface.

given by the latitude and longitude. Latitude φ_A is the angle between plane $\mathbf{O}x'y'$ (equator) and direction to point $\mathbf{A}$. Longitude λ_A is defined as an angle in the $\mathbf{O}x'y'$ plane between the zero meridian and point $\mathbf{A}$ meridian. The zero meridian and point $\mathbf{A}$ rotate in block about axis $\mathbf{O}z'$. Designate the distance between the Earth mass center $\mathbf{O}$ and point $\mathbf{A}$ as R_A. This distance is a function of latitude and parameters of the Earth's ellipsoid of revolution:

$$R_A = R_A(\varphi_A, e_{terra}, a_{terra}) \ . \tag{3}$$

In the fixed frame of reference $\mathbf{O}x^e y^e z^e$ (Fig. 3), point $\mathbf{A}$ can be defined via the position vector:

$$\underline{r}_A(t) = R_A \cdot \mathbf{P}_x(\varepsilon) \cdot \mathbf{P}_z\big(\lambda(t)\big) \cdot \begin{pmatrix} \cos\varphi_A \cos\lambda_A \\ \cos\varphi_A \sin\lambda_A \\ \sin\varphi_A \end{pmatrix} \ , \quad |\underline{r}_A(t)| = R_A \ , \tag{4}$$

where $\mathbf{P}_x$, $\mathbf{P}_z$ are the Earth rotation matrices[4]; $\mathbf{A}^e$ is the point $\mathbf{A}$ projection to the ecliptic plane $\mathbf{O}x^e y^e$; $\lambda(t)$ is the angle in the ecliptic plane $\mathbf{O}x^e y^e$ between axis $\mathbf{O}x^e$ and line passing through points $\mathbf{O}$ and $\mathbf{A}^e$. Let us define angle $\lambda(t)$ via the Sun longitude λ_{sun} for the appropriate epoch [7] and time moment selected for detecting the physical phenomenon under study, e.g., when the event referred to as "local midnight" takes place at point $\mathbf{A}$:

$$\lambda(t) = \lambda_{sun}(t) - \pi \,. \tag{5}$$

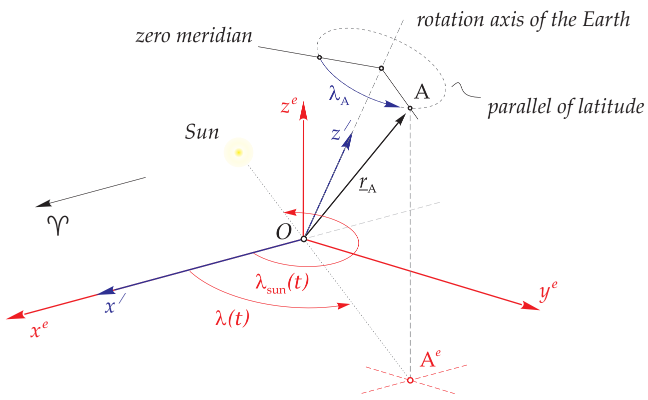

Figure 3. "Local midnight" at point $\mathbf{A}$.

Notice that the conventional *solar day* (the time interval between the Sun transits) is in fact formed by summing two angular velocities: that of

[4]Let position vector $\underline{r}$ define the point M location in the $\mathbf{O}xyz$ frame of reference; then, if we consider point M in a new frame of reference $\mathbf{O}x'y'z'$ obtained by a series of counterclockwise turns by appropriate angles, its coordinates will be different. Matrices of rotation by angle ξ about axes $\mathbf{O}x$, $\mathbf{O}y$, $\mathbf{O}z$ are given below. The counterclockwise rotation is assumed to be positive.

$$\underline{r}' = \mathbf{P}_x(\xi)\underline{r}\,, \quad \mathbf{P}_x(\xi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\xi) & -\sin(\xi) \\ 0 & \sin(\xi) & \cos(\xi) \end{pmatrix}\,, \quad \underline{r} = \mathbf{P}_x^{-1}(\xi)\underline{r}'$$

$$\underline{r}' = \mathbf{P}_y(\xi)\underline{r}\,, \quad \mathbf{P}_y(\xi) = \begin{pmatrix} \cos(\xi) & 0 & -\sin(\xi) \\ 0 & 1 & 0 \\ \sin(\xi) & 0 & \cos(\xi) \end{pmatrix}\,, \quad \underline{r} = \mathbf{P}_y^{-1}(\xi)\underline{r}'$$

$$\underline{r}' = \mathbf{P}_z(\xi)\underline{r}\,, \quad \mathbf{P}_z(\xi) = \begin{pmatrix} \cos(\xi) & \sin(\xi) & 0 \\ -\sin(\xi) & \cos(\xi) & 0 \\ 0 & 0 & 1 \end{pmatrix}\,, \quad \underline{r} = \mathbf{P}_z^{-1}(\xi)\underline{r}'$$

the Earth self rotation and that of additional rotation caused by the Earth annual revolution about the Sun:

$$\omega(t) = \omega_{terra}(t) + \omega_{year}(t) \ , \tag{6}$$

here $\omega_{terra}(t)$ is the Earth self rotation angular velocity (the Earth makes a turn around its axis with respect to stars during the time equal to $\approx 23^{h}56^{m}$); $\omega_{year}(t)$ is the additional angular rotation velocity ensuring the usual apparent alternation of sunsets and dawns, namely, the solar day. Since the measurement is performed only once a day, we can neglect the "fast" component of the Earth daily rotation ω_{terra}; after that, the Earth rotation speed will be represented only by the additional angular velocity $\omega_{year}(t)$. This additional angular velocity is the time derivative of the Sun longitude $\lambda_{sun}(t)$, thus (6) gets the following form:

$$\omega(t) = \omega_{year}(t) = \frac{d\lambda_{sun}(t)}{dt} \ , \quad where \qquad \frac{2\pi}{\omega_{year}(t)} \approx 365 \ day \tag{7}$$

This means that we assume the Earth to move round the Sun in the space facing it always with the same side. Accordingly, the Observer staying at any point chosen on the Earth surface will always retain his position with respect to the direction to the Sun.

4. Moon

The Moon's orbit is a complex open spatial curve. The Moon motion is considered with respect to fixed point $\mathbf{O}$ coinciding with the Earth mass center (Fig. 4). The Moon location in the $\mathbf{O}x^e y^e z^e$ frame of reference is set by a combination of 6 cyclically variable orbit components [2].

$$\underline{r}_{lune}(t) = \underline{r}_{lune}\big(\, i(t), \psi(t), \varphi(t), e(t), a(t), t_*(t) \,\big) \ , \tag{8}$$

where $i(t)$ is the orbit tilt angle defined as the angle of intersection between the ecliptic plane $\mathbf{O}x^e y^e$ and the Moon's trajectory plane; $\psi(t)$ is the node line longitude (the inter-plane intersection line); the nodes are counted from the $\mathbf{O}x^e$ axis that is parallel to the vernal equinox line Υ at any moment of the Earth motion (point $\mathbf{O}$) about the Sun; $\varphi(t)$ is the angle between the nodal line and line of apsides; the Moon's trajectory ellipticity is defined by eccentricity $e(t)$ and semimajor axis $a(t)$; t_* is the time moment when the Moon passes through the perigee point. For instance, relations for $\psi(t)$ and

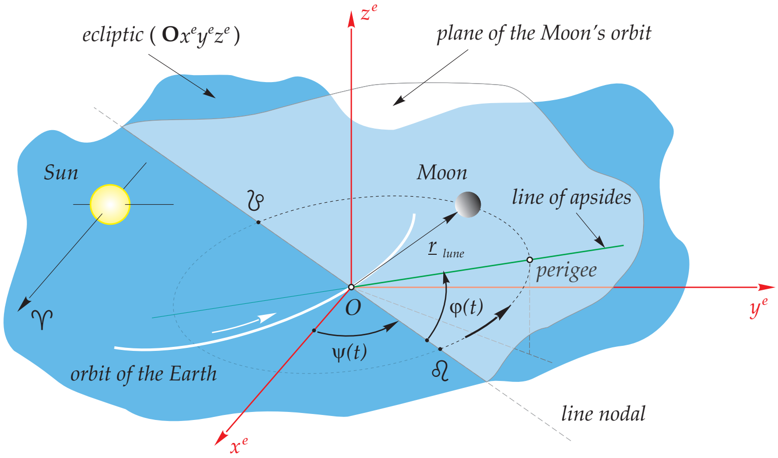

Figure 4. Moon's orbit components.

$\varphi(t)$ taken from paper [2] for the 1900 epoch look like:

$$\begin{aligned}
\psi(t) &= 259^\circ\,10'\,59''\!.77 - 1934^\circ\,08'\,31''\!.23 \cdot \tau + 07''\!.48 \cdot \tau^2 + 0''\!.0080 \cdot \tau^3 \,, \\
\varphi(t) &= 75^\circ\,08'\,46''\!.61 + 6003^\circ\,10'\,33''\!.75 \cdot \tau - 44''\!.65 \cdot \tau^2 - 0''\!.0530 \cdot \tau^3 \,, \\
& \qquad\qquad \tau(t) = (2415020 - t)/36525 \,,
\end{aligned} \tag{9}$$

where $\tau(t)$ is the time expressed in Julian centuries as a function of the Julian date t. Derivatives of these functions give us the periods of rotation of the nodal line and line of apsides, respectively:

$$T_\psi = \frac{2\pi}{{}^{d\psi}\!/_{dt}} \approx -18.6 \text{ year} \,, \qquad T_\varphi = \frac{2\pi}{{}^{d\varphi}\!/_{dt}} \approx 6 \text{ year} \tag{10}$$

Therefore, according to the angular velocity sum rule, rotation period of the perigee involved in these two rotations about point $\mathbf{O}$ in the $\mathbf{O}x^ey^ez^e$ frame of reference is

$$T_{perigee} = \frac{T_\psi \cdot T_\varphi}{T_\psi + T_\varphi} \approx 8.8 \text{ year.} \tag{11}$$

Moon's perigee mass. Let us find out how the Moon's perigee displacement affects the direction and magnitude of gravitational acceleration at point **A** on the Earth's surface. Substitute the Moon's gravitational effect on the Earth with the equivalent gravitational effect of a certain body located in the Moon's perigee. Derive this body mass from the Moon's gravitational effect on the fixed Earth (point $\mathbf{O}$) during one cycle $T_{lune} \approx 28$ days. Due to the axial symmetry and non-zero eccentricity, the resulting force of the gravitational nature will be directed towards the perigee (Fig. 5). The force

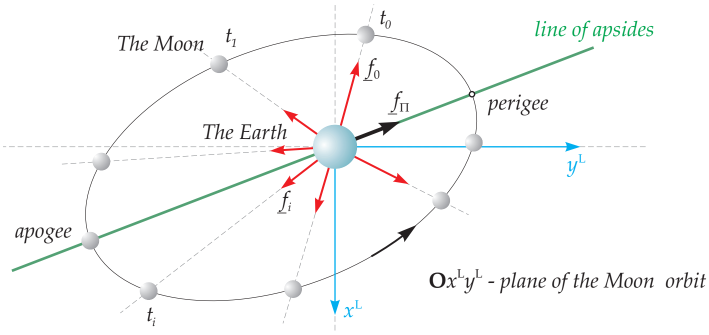

Figure 5. Force $\underline{f}_\Pi$ is the Moon's gravitational impact on the Earth for the period of its revolution about the Earth by the Keplerian orbit.

module $|\underline{f}_\Pi|$ is the integral of the Moon's gravitational impact on the Earth over the time interval equal to the period of the Moon revolution about the Earth:

$$|\underline{f}_\Pi| = \frac{1}{2\pi} G M^*_{terra} M_{lune} \int_0^{2\pi} \frac{\cos\alpha}{r(\alpha)^2}\, d\alpha\ , \quad M^*_{terra} = M_{terra} + M_{lune}\ , \tag{12}$$

where

$$r(\alpha) = \frac{p}{1 + e\cos\alpha}\ , \quad p = a(1 - e^2)\ , \quad \alpha = \frac{2\pi}{T_{lune}}\, t\ . \tag{13}$$

Here $r(\alpha)$ is the Moon's focal radius as a function of angle α counted counterclockwise from the direction to the perigee; p is the focal parameter; e is the eccentricity; a is the semimajor axis. Integrating (12), we obtain:

$$|\underline{f}_\Pi| = G M^*_{terra} M_{lune} \frac{e}{p^2}\ . \tag{14}$$

Now, knowing force module $|\underline{f}_\Pi|$, we can write a relation for a certain mass ensuring the necessary gravitational effect. Hereinafter we will refer to this mass as the *Moon's perigee mass*. Let us derive the desired mass from the two-body gravitational interaction law:

$$m_\Pi = \frac{|\underline{f}_\Pi|}{G M^*_{terra}} \Pi^2_{lune}\ , \quad \Pi_{lune} = \frac{p}{1+e}\ , \tag{15}$$

here Π_{lune} is the distance between the ellipse focus (point **O**) and the perigee. Substituting (14) into (15), we obtain the expression for the perigee mass:

$$m_\Pi(e) = M_{lune} \frac{e}{(1+e)^2}\ , \quad m_\Pi(e)\Big|_{e=0} = 0\ . \tag{16}$$

Thus, the Moon's perigee mass is defined as a function of the Moon's Keplerian orbit eccentricity. As mentioned above, the Moon's orbit components are of the cyclic character; therefore, we can assume in the first approximation that eccentricity $e(t)$ is a harmonic function with the period equal to the time of the perigee revolution about the Earth center [2]:

$$e(t) = \overline{e} + \frac{1}{2}(e_{max} - e_{min}) \sin\left(\frac{2\pi}{T_{perigee}} t\right) , \quad \overline{e} = \mathrm{const} . \tag{17}$$

Introduction of dummy perigee mass $m_{\Pi}(e)$ allowed us to exclude the "fast" Moon motion component and consider only the Moon's perigee motion. The perigee mass position in the fixed frame of reference $\mathbf{O}x^e y^e z^e$ is determined by position vector:

$$\underline{r}_{\Pi}(t) = \overbrace{\Pi_{lune}(e, a)}^{a(1-e)} \cdot \mathbf{P}_z(\psi(t)) \cdot \mathbf{P}_x(i(t)) \cdot \mathbf{P}_z(\varphi(t)) \cdot \underline{e}_1 \tag{18}$$

5. Plumb-in line and Moon's perigee

Let us reveal how angle $\gamma(t)$ depends on the mutual arrangement of the Moon's perigee $\mathbf{\Pi}$ and Observer $\mathbf{A}$ on the rotating Earth surface (Fig. 6).

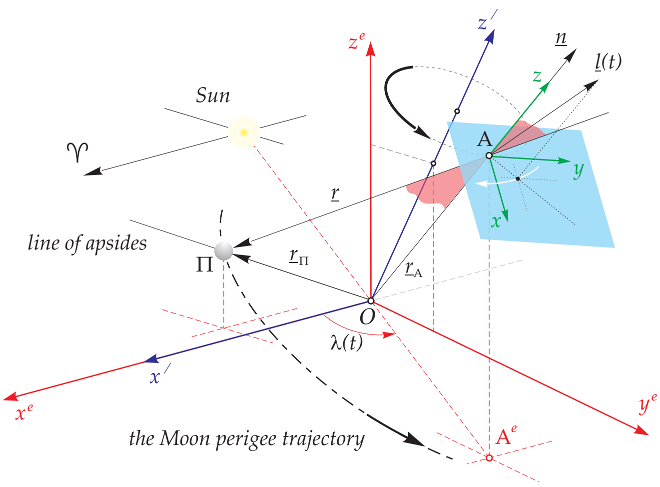

Figure 6. The Moon's perigee $\mathbf{\Pi}$ and Observer $\mathbf{A}$ in the fixed frame of reference $\mathbf{O}x^e y^e z^e$.

In the $\mathbf{O}x^e y^e z^e$ frame of reference, point $\mathbf{A}$ (Observer) and point $\mathbf{\Pi}$ (Moon's perigee mass) rotate along their trajectories about point $\mathbf{O}$ (the Earth's mass

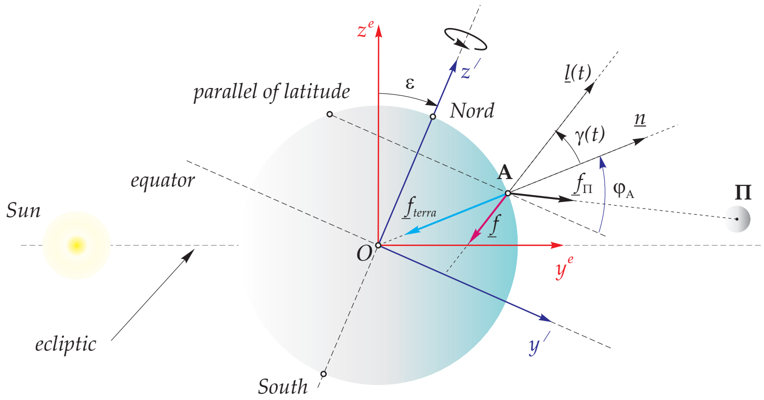

Figure 7. Forces acting on the point mass at point **A** at the moment when the Sun, Earth and Moon's perigee are in $\mathbf{O}y^e z^e$.

center) in the same direction (counterclockwise) but with different angular velocities. Point mass $m_A = 1$ at point **A** (Fig. 7) is subject to two gravitational forces (from the Earth and Moon's perigee mass).

$$\underline{f}_{terra}(t) = \mathbf{G}\, m_A M_{terra} \frac{\underline{r}_A}{|\underline{r}_A|^3}\ , \quad \underline{f}_\Pi(t) = \mathbf{G}\, m_A m_\Pi \frac{\underline{r}_\Pi - \underline{r}_A}{|\underline{r}_\Pi - \underline{r}_A|^3}\ , \tag{19}$$

here $\underline{r}_\Pi(t)$, $\underline{r}_A(t)$ are the Moon's perigee (18) and observer's (4) position vectors, respectively. Designate as $\underline{f}(t)$ the sum of forces acting at point **A**:

$$\underline{f}(t) = \underline{f}_{terra}(t) + \underline{f}_\Pi(t)\ , \tag{20}$$

in this case, force $\underline{f}(t)$ in the mobile frame of reference $\mathbf{A}xyz$ is

$$\underline{f}_A(t) = \mathbf{P}_A(t)\cdot\underline{f}(t)\ , \quad |\underline{f}_A(t)| = |\underline{f}(t)|\ , \tag{21}$$

where

$$\mathbf{P}_A(t) = \mathbf{P}_x(-\varepsilon)\cdot\mathbf{P}_z\left(\lambda(t)\right)\cdot\mathbf{P}_y\left(\frac{\pi}{2} - \varphi_A\right)\ . \tag{22}$$

Computations showed that the vector $\underline{l}(t)$ apex in the $\mathbf{A}xyz$ frame of reference (Fig. 6) moves cyclically counterclockwise about normal $\underline{n}$ with the period of $T_{cycle} \approx 411.8$ days. Figs. 8 and 9 illustrate this process graphically. Period $T_{cycle} \approx 411.8\ days$ in the $\mathbf{A}xyz$ rotating frame of reference is a result of summing two rotations: the Observer's (point **A**) rotation about axis $\mathbf{O}z'$ with a period equal to that of the Earth revolution about the Sun and Moon's perigee rotation about the Earth's center. The long-period (≈ 8.8 year) component of the observed process (Fig. 8) is determined only by cyclic variations in the Moon's elliptic trajectory eccentricity and semimajor axis. Some differ-

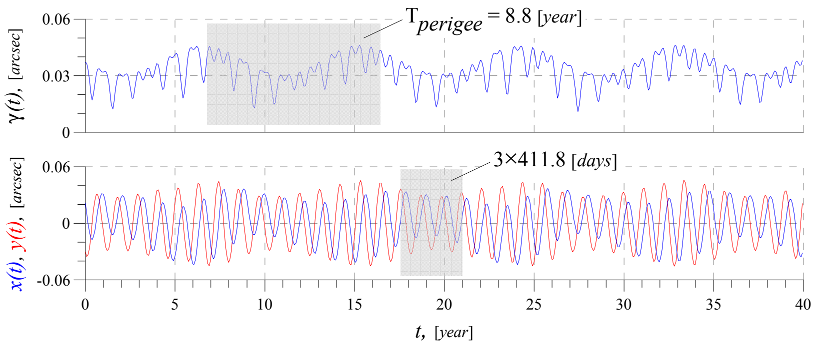

Figure 8. Angle $\gamma(t)$ as a function of the Moon's perigee motion.

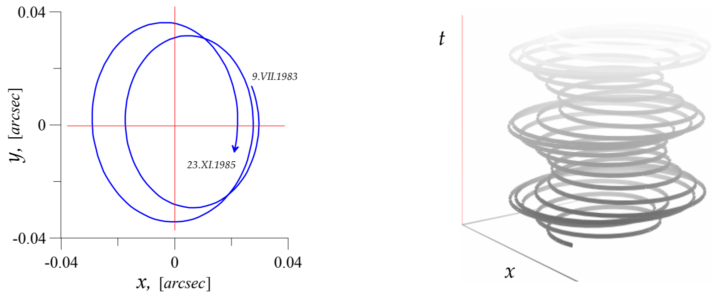

Figure 9. A fragment of the vector $\underline{l}(t)$ apex trajectory projection on plane $\mathbf{A}xy$ and its time scan.

ence in the amplitudes is inevitable since the data acquisition and processing techniques used in astrometry are based on the following postulate: *variations in the point latitude are caused by motion of the Earth instant rotation axis (Chandler wobble) in the absolute absence of external forces.* Actually, each latitude measurement with classical astrometric instruments occurs in the gravitational field that varies continuously due to the Earth's self-rotation and motion relatively to the Sun, Moon and planets. Naturally, continuous and non-random variations in the gravitational pattern at the observation point on the Earth change the spatial attitude of the plumb-in line (or artificial horizon) [4, 1]. This means that it is impermissible to combine in one time series observations obtained with different instruments under different gravitational conditions.

The physical process suggested here is the only one that explains the field line spatial variations with the period of about $\approx$411.8 day. Numerical modelling of the process fit well the results of long-term observations of variations in the Earth gravitational field $\mathbf{\Delta g}$ performed at the *Bad Homburg* and *Boulder* stations (Fig. 10). The *Johannesburg* and *Brussels* observations of the

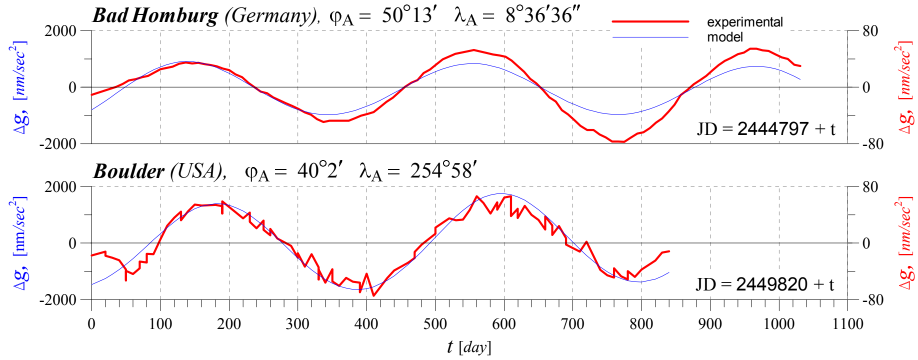

Figure 10. Comparison of the computed and actual variations in gravitational acceleration $\mathbf{\Delta g}$ at stations *Bad Homburg* [11] and *Boulder* [12].

"variation of latitude" $\Delta\varphi(t)$ (Fig. 11) also confirm our version of the Chandler wobble nature and the actual absence of the Chandler's residual motion of the Earth instant rotation axis within the Earth. It should be emphasized that the observed "variation of latitudes" (more exactly, the variation in the field line deviation angle) and the gravitational acceleration variation have the same period and are strictly antiphased.

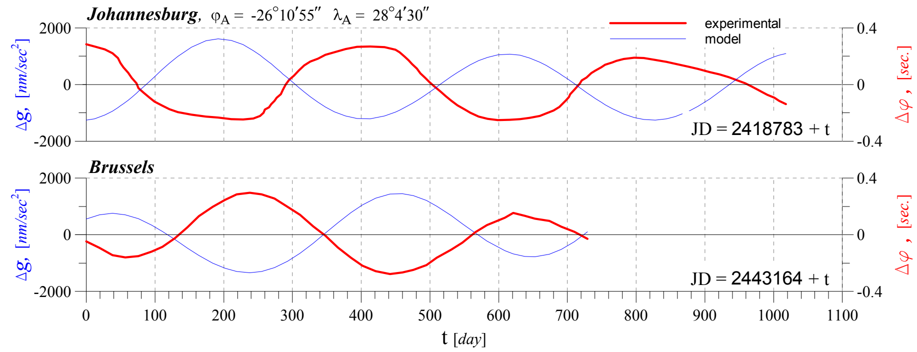

Figure 11. Comparison of variations in the gravitational acceleration and "variation of latitudes" at stations *Johannesburg* [8] and *Brussels* [3].

The existence of the Chandler wobble (with the approximately ≈411-day period) has been confirmed by analyzing cycle duration (Fig. 12) of the so called residual (or "Chandler") motion of the Earth rotation axis based on the `IERS` data[5]. Among the entire totality of periods revealed, only the ≈411-day period reflects the really existing process, i.e., is of the natural origin. Other periods, including the statistical pseudo-period of ≈432 days, do not reflect

[5] `http://hpiers.obspm.fr/eop-pc/`

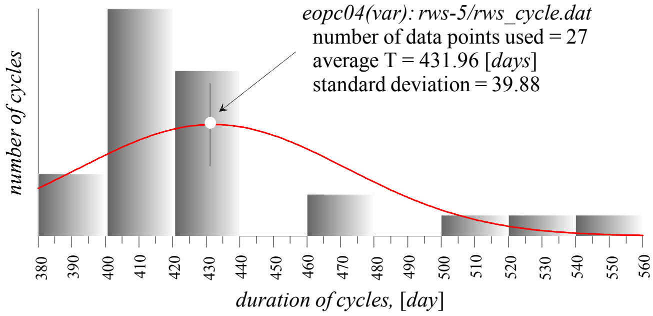

Figure 12. Distribution of the hodograph cycles duration [6].

actual phenomena but result from drawbacks of the experimental technique used in observation.

Fig. 13 illustrates the Moon influence on the variation in the spatial attitude of the Earth–Moon system gravitational field line and, hence, in deviation of tangent vector $\underline{\mathfrak{l}}(\mathtt{t})$. When the external gravitating mass approaches,

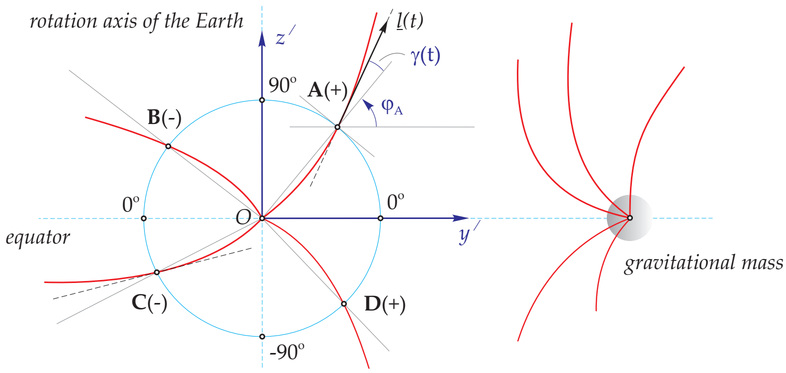

Figure 13. Plumb-in line $\underline{\mathfrak{l}}(\mathtt{t})$ deviation at different Earth surface points as a function of an external gravitating mass.

an imaginary effect of the latitude increase takes place at points **A** and **D**, while latitudes at points **C** and **B** seem to decrease. Therefore, when the star zenith distances are observed simultaneously at one and the same meridian but on different sides of the equatorial zone, the plumb-in line $\underline{\mathfrak{l}}(\mathtt{t})$ deviations shall be in-phase.

Applying the resonance method of analyzing irregular time series [5] to perennial observations of the Earth gravitational field variations obtained in the scope of the `OHPDMC`[6], we confirmed the existence of the gravitational field perturbations with the characteristic period of about ≈411 days. Let us

[6]`http://ohp-ju.eri.u-tokyo.ac.jp/`

refer to this period according to its historical proper name *Chandler's*; here we should add that Chandler wobble can be detected in every gravitation-based process observed on the Earth surface. Magnitudes of such "Chandler wobbles" will always depend on the actual parameters of the earth Observer motion with respect to the Moon's perigee.

6. Conclusions

- In this paper we state that such a theoretically revealed phenomenon as Chandler wobble[7] (residual motion of the Earth rotation axis within the Earth) does not exist in reality. The observed variations in the star zenith distance measurements are caused by variations in the plumb-in line that is used as a measuring instrument reference. The plumb-in line (gravitational acceleration vector) deflections occur due to the gravitating bodies surrounding the Earth, e.g., the Moon.

- Wrong interpretation of the zenith distance variations shown by astrometric instruments required for corrections *for the pole displacement* and *for irregularity of the Earth rotation* that are not only useless but, moreover, deform the verity. These corrections distort the Universal Time system, navigation systems (`GPS`, `GLONASS`, etc.), results of geodesic, metrological and physical measurements, etc.

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[7] Let us refer to the 411-day period according to its historical proper name *Chandler's*; note that Chandler's period can be detected in gravitation-based processes observed on the Earth surface. Magnitudes of such period of Chandler will always depend on actual parameters of the earth Observer–Moon mutual motion.

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